NASA TN D-134

7N-13
197152
40P

NASA TN D-134

TECHNICAL NOTE

D-134

SOME THRUST AND TRAJECTORY CONSIDERATIONS FOR LUNAR LANDINGS

By Richard J. Weber and Werner M. Pauson

Lewis Research Center
Cleveland, Ohio

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

WASHINGTON November 1959

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

TECHNICAL NOTE D-134

SOME THRUST AND TRAJECTORY CONSIDERATIONS FOR LUNAR LANDINGS

By Richard J. Weber and Werner M. Pauson

SUMMARY

A proposed method for accomplishing soft landings on the moon is first to establish a circumlunar orbit and then to transfer to the lowest acceptable altitude by a minimum-energy elliptical path. After braking to a halt, a vertical descent is made consisting of free fall and a final upward thrust application to decelerate the vehicle. The characteristic-velocity increment ΔV for a typical landing (starting from orbit) is 6880 feet per second.

There is little reduction in ΔV to be gained during either perigee deceleration or vertical descent by using thrust-mass ratios in excess of 2 moon g's. Also, the impact velocity is more sensitive to engine starting errors during vertical descent for high thrust-weight ratios. If errors can be held sufficiently small, constant-thrust engines may be adequate. However, a two-to-one thrust variation is preferable, and as little as 10-percent variability is helpful.

INTRODUCTION

An artificial satellite or a spaceship that attempts to land on the surface of the Earth encounters the difficulties of atmospheric reentry, which are the subject of much current interest and study. A spaceship seeking to land on a planet without an atmosphere will escape these difficulties but, in turn, must forego the benefits afforded by an atmosphere: (1) deceleration without energy expenditure, and (2) controlled utilization of aerodynamic forces for maneuverability. Landing techniques are obviously very different for the two cases.

Many references to lunar landings are found in the literature - for example, references 1 to 7. However, most of the published information is qualitative only. Reference 4 presents a study of landings of an unmanned instrument carrier at moderate to high impact speeds, with emphasis on the nature of the actual impact. Reference 5 is an interesting description of a hydraulic analog for simulating a spaceship falling vertically toward the moon.

A type of flight trajectory for the landing of manned vehicles on the moon is discussed herein. For this nominal trajectory, emphasis is placed on the factors influencing selection of the engine thrust level. Calculations are made of the velocity increments required for each phase of the landing. The sensitivity to various kinds of errors is studied.

In the analysis the vehicle is treated as a point mass; that is, rotation about its center of gravity is not considered.

ANALYSIS AND RESULTS

Lunar Characteristics

For the purposes of this study, the moon is considered to be a nonrotating homogeneous sphere of radius 940 nautical miles, with an acceleration due to gravity of 5.32 feet per second per second at the surface. In actuality, the moon is believed to bulge in the direction of the Earth by about 1 mile, and its equatorial speed of rotation is 15 feet per second. These attributes must be considered in precision calculations but, it was felt, could be neglected in the present analysis.

Geographically, the lunar surface is quite irregular, with some mountains as high as 25,000 to 30,000 feet. More generally, however, the heights of the mountains and crater walls are not greater than 5000 to 10,000 feet. These heights are significant insofar as they restrict horizontal motions during landing maneuvers.

The fine details of the surface that make any particular point suitable for a landing site are of great importance but are not within the scope of this study. (Such factors have been discussed in refs. 1 and 6, e.g.) Of course, the most important characteristic of the moon with respect to landing procedures is that it possesses no significant atmosphere and hence gives rise to no aerodynamic forces.

Basic Flight Path

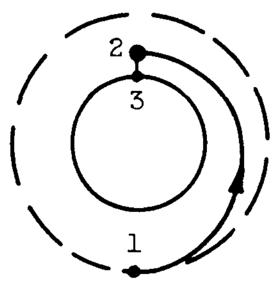
In general, a spaceship will approach the vicinity of the moon with hyperbolic flight velocity (relative to the moon). It is probably quite feasible to have this hyperbolic course intercept the moon, so that the vehicle moves along a more or less vertical path with respect to the lunar surface, and to accomplish a direct landing after applying one or more bursts of retrothrust. This is the procedure described in reference 1, for example. An alternative procedure is to let the hyperbolic path miss the moon and, at about the point of closest approach, bring the vehicle into a lunar orbit from which the final landing would be made. Possible advantages of this procedure are (1) errors in the hyperbolic trajectory will affect the lunar orbit that is established but will not necessarily cause a displacement from the desired landing point, as would be the case for the direct-landing method; and (2) there is an opportunity to remain in orbit long enough to determine precisely the vehicle's position and velocity, to inspect the lunar surface, select a landing point, and plan the descent. Thus, as suggested in reference 7, greater assurance is gained of being able to achieve a landing at any particular desired point.

This second procedure has been adopted in the present study where, for the sake of convenience, the orbit has been assumed to be circular. (The means of establishing this orbit is not considered herein.) In the first part of the study it is further assumed that the desired landing point lies on the lunar great circle contained by the orbital plane.

Landings with a horizontal velocity component were felt to be undesirable because of the irregular surface of the moon. The nominal requirement was therefore set that only vertical flight is permitted near the surface. In actuality, of course, some horizontal motion will almost certainly be present. The acceptable magnitude of horizontal velocity will depend on the relative difficulties of designing the guidance to reduce the velocity and designing the vehicle to tolerate higher velocity. The problems of achieving small horizontal velocity at impact are not treated in this report.

<u>Description of flight path</u>. - A simple concept for performing the landing is to reduce the spaceship velocity to zero at the moment of passing over the landing point. The spaceship then falls vertically under the influence of gravity. An instant before the vehicle strikes the ground, a second impulse is applied to reduce the velocity again to zero. This is an idealized procedure in that only a finite amount of thrust is available, and so the velocity reductions cannot be made impulsively.

Even for this idealized impulsive case, a vertical fall is an inefficient way (i.e., costly in propellant consumption) to lower the spaceship altitude, particularly since uncertainties in the hyperbolic approach trajectory may cause the initial orbit to be quite high. It is more efficient to transfer first to a lower-altitude orbit before commencing the final vertical descent. A cotangent (or Hohmann) ellipse is the most efficient transfer process. Because the Hohmann transfer time is only in the order of an hour, the more rapid excess-energy paths have not been considered (although an argument might be made for the excess-energy paths in that the descent could be carried out within sight of the landing point; the vehicle could then be tracked during descent by the landing station or even from Earth if the landing point is on the near



(a)

side of the moon). The entire flight path is indicated in sketch (a). At point 1, the antipode of the selected landing position, a small tangential retrothrust is applied to the vehicle, which then descends along the elliptical path 1-2. At point 2, the perigee[1] of the ellipse, a second retroimpulse reduces the velocity to zero. The vehicle then falls vertically, with a third decelerating impulse applied just before impact at point 3.

The previously mentioned all-vertical descent is seen to be a special case of this more general maneuver.

Perigee altitude. - The characteristic-velocity increment ΔV for accomplishing the landing by means of impulsive thrust is shown in figure 1 for various perigee altitudes. This ΔV is used as a measure of the required propellant consumption, being defined as

$$\Delta V = I g_E \ln \frac{W_i}{W_i - W_p}$$

(Symbols are defined in appendix A.) For the idealized impulsive case, ΔV is equal to the actual velocity change; for nonimpulsive cases, ΔV is often substantially greater.

The right end of the curves in figure 1 represents direct vertical descent from the initial orbit. Significant reductions in ΔV can be achieved by transferring first to a lower altitude before undertaking the final vertical descent (unless, of course, the original altitude is already low). For example, when starting from a 500,000-foot orbit, a saving in ΔV of 1660 feet per second can be made by transferring to a perigee altitude of 10,000 feet. The greatest saving is made when the perigee altitude is set at zero; that is, the vertical fall is eliminated altogether. This, however, violates the requirement of only vertical flight in the vicinity of the surface.

Minimum-energy elliptical transfer to a lower altitude followed by vertical descent is therefore selected as the basic landing maneuver. The succeeding sections investigate this procedure in more detail, taking into account possible trajectory errors and nonimpulsive retrothrust. The equations employed to calculate the results are derived in appendix B.

---

[1]For the sake of brevity, the upper and lower apsides of the circumlunar orbit are designated apogee and perigee.

Departure from Orbit

To initiate the departure from the circular orbit, retrothrust is applied to the vehicle tangential to the flight direction. This transfers the vehicle to an ellipse, the perigee altitude of which depends on the magnitude of the retarding impulse. As a typical example, if the initial altitude is 500,000 feet (81 naut. miles) and the desired perigee altitude 10,000 feet, then the required impulsive ΔV_1 is 109.8 feet per second. For any F/W_m greater than about 1/4, this small velocity change can be accomplished with negligible altitude loss; therefore, no nonimpulsive correction need be made here. (Note that weights given in this report are referenced to the lunar surface; therefore, the quoted values of thrust-weight ratio are in terms of lunar g's. To convert to Earth g's, the given values should be multiplied by 1/6.)

Even though ΔV_1 is a small quantity, the perigee altitude of the transfer ellipse is very sensitive to errors in the magnitude of ΔV_1. For instance, if the initial altitude is 500,000 feet, each error of 1 foot per second in ΔV_1 alters the perigee altitude by about 4500 feet. Thus an excess of slightly more than 2 feet per second would cause the vehicle in the previous example to crash. Errors in thrust alinement will raise the perigee altitude and are therefore not as critical as errors in the magnitude of ΔV_1. Also, the perigee altitude is not as sensitive to errors in thrust alinement as it is to errors in ΔV_1. For example, a thrust-alinement error of $\pm 10^\circ$, which is rather large, causes an increase of only about 6000 feet in the perigee altitude.

With a fixed-thrust engine, the magnitude of the ΔV is determined only by the duration of firing. Sensitivity to duration errors and to the effects of starting and shutdown transients, if significant, can be minimized by selecting a long firing duration. This requires a low F/W_m.

The preceding discussion presupposes that the elements of the initial orbit are known perfectly. Actually, uncertainties in determining the orbit will exist and lead to errors at apogee even if ΔV_1 is applied exactly as planned.

In the preceding sections it was observed that the perigee altitude should be high enough to avoid interference by surface projections. That altitude must now be increased by an amount adequate to tolerate the expected errors during the apogee maneuver.

Perigee Maneuver

Upon reaching perigee the vehicle must be decelerated to zero horizontal velocity. Since the velocity is somewhat greater than the circular orbital value, the required ΔV is large. As a numerical example, consider descent from a 500,000-foot orbit to a 50,000-foot perigee (acknowledging the indication of the previous section that the perigee should not be too low). For this case the impulsive ΔV_2 is 5586 feet per second. Because of the magnitude of this value and the fact that the velocity is being reduced below the orbital speed, it is necessary to investigate in more detail the nonimpulsive motion of the vehicle.

E-527

In order to simplify the analysis, results for the nonimpulsive cases were calculated on the basis of holding the thrust-weight ratio constant during each maneuver. Since the consumption of fuel causes a reduction in weight, this assumption requires the thrust to be varied in the same proportion. In practive, however, it is easiest to hold the thrust constant, in which case the presented results still apply if F/W_m is interpreted as an average value for the maneuver.

Horizontal deceleration. - If the vehicle is spin-stabilized, the simplest method of deceleration, with respect to guidance, is to leave the vehicle and the thrust vector in a constant attitude until motion is halted. Although such stabilization is probably not feasible for manned vehicles, calculations were made for constant-horizontal-attitude firing. (The equations were actually derived for circumferentially directed thrust, but the stopping distance is so short that this corresponds to constant-attitude firing.)

As the vehicle slows down, centrifugal force is insufficient to maintain altitude, and so the vehicle starts to fall. By the time horizontal deceleration is completed, the vehicle is lower than the selected perigee point and it has some vertical velocity. This is illustrated in figure 2 as a function of the thrust-weight ratio. The variation in ΔV_2 is not plotted, since it differs only slightly from the impulsive value for F/W_m greater than 2.

The loss in altitude is very appreciable at the lower thrust levels. In fact, for values of F/W_m less than 5.3, the altitude loss is greater than 50,000 feet, which means that the vehicle would strike the ground before the horizontal deceleration is completed. The large altitude loss that occurs with intermediate values of F/W_m, between 6 and 10, say, would not be too serious if the assured perigee altitude were really 50,000 feet above the surface. However, this is merely a nominal value, since, as previously pointed out, errors in the apogee maneuver coupled with the proximity of nearby mountains might reduce the available maneuvering altitude to considerably less than 50,000 feet. In order to allow sufficient time and altitude for the terminal vertical deceleration, it appears that this type of perigee deceleration should be accomplished with a large F/W_m, in the order of 10 or more (i.e., about 2 Earth g's).

The vertical velocity at the conclusion of the maneuver is substantial (fig. 2(b)). However, it is of about the same magnitude that would exist had the vehicle been halted impulsively at perigee and then fallen under gravity through the same decrease in altitude. The amount of braking at the end of free fall (i.e., before impact) is therefore little affected by the value of F/W_m for this method of perigee deceleration.

Another factor that is significant is that considerable horizontal distance is traveled while the vehicle is being decelerated. This must be allowed for if a particular landing spot is desired.

Constant-altitude deceleration. - The use of horizontal thrust requires a relatively high thrust-weight ratio in order to limit the altitude loss during deceleration. This high F/W_m is undesirable, since a low-thrust engine is easier to develop and is more consistent with the requirements of other phases of the landing. Some improvement can be made by rotating the engine during deceleration so that the thrust is alined with the velocity vector. This procedure is identified by the terms "tangential firing" or "gravity turn." Because tangential firing yields an upward component of thrust, the altitude loss is reduced to about half that suffered with horizontal firing at each value of F/W_m.

A third method of deceleration is possible that eliminates the altitude loss entirely. With a constant-thrust engine this can be done by continuously rotating the thrust vector so that its vertical component increases by the same amount that the centrifugal force of the slowing vehicle decreases. (This method is described in greater detail in appendix B.) The ΔV requirements for this procedure are given in figure 3, which shows that there is little penalty in using F/W_m ratios as small as 3, or possibly lower. The circumferential displacement during perigee deceleration is also shown in figure 3 for this method. The displacement is quite large, although little higher than would exist for the case of horizontal deceleration had such low thrust-weight ratios been used.

Vertical Descent

After halting the vehicle at perigee, free vertical fall takes place. The velocity gained during the fall (plus any vertical velocity existing after perigee deceleration) must then be dissipated by engine thrust. The engine must be started at an altitude such that the velocity is reduced to zero at the moment of touchdown.

Effect of thrust-weight ratio. - Figure 4 shows the elapsed time, ΔV, and thrust-initiation altitude for two perigee altitudes as a function of F/W_m. If the thrust were infinite, it could be applied for an infinitesimally short length of time an instant before striking the ground. With lower values of thrust, the engine must be started sooner and applied longer.

In the limiting case, the engine thrust is kept equal to the vehicle weight and is applied as soon as a desired value of sinking speed is reached (50 ft/sec after 9.4 sec of fall for the point in fig. 4). Keeping the thrust on continuously in this fashion probably renders the vehicle more controllable and undoubtedly contributes to the pilot's peace of mind. However, the ΔV for this procedure is too high to be accepted. Free fall followed by a terminal constant-thrust braking appears to be a more feasible method of vertical descent. Any F/W_m greater than about 2 is satisfactory on the basis of ΔV. A reliable, rapidly starting engine is obviously essential. It is unlikely that this critical vertical descent can, in practice, be accomplished exactly according to plan. An investigation was therefore made to determine the sensitivity of the descent to various errors in applying the final braking thrust.

Starting error. - Once the engine thrust level is fixed, then, after the vehicle begins to fall from a given perigee altitude, there is only one instant (or one altitude) at which the engine may be started in order to have reduced the vertical velocity to zero at the moment the ground is touched. Figure 5 shows the effect of starting errors for several F/W_m values. If the thrust is applied too late, the vehicle will have insufficient time to slow down and will strike the ground with finite speed. Conversely, if thrust is applied too soon, the vehicle will achieve zero velocity while still some distance above the ground. It is assumed that the thrust is terminated at this condition, and the vehicle then falls and hits the ground. (The use of thrust modulation to alleviate this situation is discussed later.)

E-527

It is seen that a given error has less serious consequences if the engine is started early rather than late. If the error is equally likely to occur in either direction, it is therefore preferable to plan to fire somewhat early in order to have equal tolerance on either side. The tolerable errors depend on how great an impact velocity the vehicle is designed to withstand.

The vehicle is less sensitive to starting errors, particularly to late starts, if F/W_m is low. This is because the error is then a smaller portion of the total burning time.

In this calculation the thrust was assumed not to vary during the descent. However, if adequate information about the descent (such as altitude and rate of descent) were supplied to the pilot or automatic controls, then the availability of thrust variation would make it possible to compensate for starting errors.

The amount of correcting thrust modulation required to attain zero impact velocity is dependent on (1) the magnitude of the starting error, (2) the design thrust level, and (3) the nature of the guidance and control system. In order to gain some insight into the required magnitude of modulation without detailed knowledge of item (3), it was simply assumed that the vehicle falls through a given distance y with the thrust at the design value and the thrust is then changed to the proper new value which is held constant until touchdown.

Figure 6(a) shows that the required modulation varies linearly with the starting error for each design thrust level. In this figure the control reaction distance y is held equal to 2000 feet. The least modulation is required for the low thrust level. This is because thrust is initiated at a higher altitude for this case and, after traveling through y, a greater distance is available over which to apply the corrected thrust.

Figure 6(b) shows the effect of varying y for a fixed time error of 1 second. (In both parts of fig. 6 the curves are drawn for early firings, but nearly identical results are obtained for late firings.) Again, because of the longer distance available for applying corrected thrust, the low-thrust case is least sensitive to variations in y. The low-thrust case requires the least thrust modulation even if y is taken as a percentage of the thrust-initiation altitude (shown by the circles) rather than a constant. Because of the crudeness of this analysis it is not possible to form a definite conclusion. The results seem to indicate, however, that starting errors may be compensated for by varying the thrust in the order of 5 to 10 percent if the design F/W_m is about 2.

If the engine is capable of providing wide thrust variations, the efficiency of a high-thrust descent can be combined with the controllability of the continuous-thrust case. In this technique, a period of free fall would be followed by a high-thrust braking impulse fired early, so that the vehicle is brought nearly to a halt while still above the surface. The thrust would then be reduced until it equalled the weight, and the vehicle would be lowered slowly the remaining distance. The ΔV penalty for this procedure is not large. For example, consider descent from 50,000 feet with F/W_m of 2. Suppose thrust is initiated at an altitude such that the vehicle is decelerated to 20 feet per second while still 1000 feet above the surface; F/W_m is then reduced to 1, and the vehicle travels the remaining distance at constant velocity. The ΔV for this case is 1248 feet per second, or only 216 feet per second higher than for the constant-thrust case. This is perhaps the most desirable landing technique, but it requires a reliable, precisely controlled engine whose thrust can be varied by at least a factor of 2.

Thrust error. - Although the engines will presumably be calibrated in advance, it is possible that a fixed-thrust engine may not deliver exactly the expected amount of thrust at the moment of use. The effect of various relative thrust errors is shown in figure 7. If the thrust is higher than was anticipated, the impact velocity is fairly sensitive to the design thrust level, being most sensitive for the low-thrust case. If the thrust is lower than anticipated, the same impact velocity results from any given percentage thrust error, regardless of the design thrust level.

A conflict occurs in the thrust requirements for minimizing vertical braking errors with fixed-thrust engines. Low thrusts were desirable when considering timing errors, while high thrusts are desirable here. It is probably easier to calibrate the engine accurately beforehand than to eliminate timing errors while landing, so that a low-thrust engine still appears preferable for the vertical descent.

Selection of Landing Point

The only mention of the landing point up to now was the assumption that it lay somewhere on the great circle directly below the original circular orbit. To land on a particular point along this great circle, it is necessary to initiate the apogee transfer maneuver at the antipode of the landing point (actually somewhat ahead of the antipode to allow for the finite deceleration times at apogee and perigee). Timing errors in initiating the thrust at either apogee or perigee will result in a displacement of the actual landing point along the great circle. Each second of error causes a displacement of about 5500 feet.

Free-fall corrections. - After completing the elliptical transfer, it is desirable to be able to change the landing point. This may be for the sake of correcting errors or to permit more precise selection of a suitable landing area. An opportunity is available to do this during the vertical descent. After the perigee deceleration and before the terminal braking maneuver, there is a period of free fall. During this period it is feasible to direct the thrust horizontally and so effect changes in position.

The maximum amount of horizontal motion that can be achieved is shown in figure 8. Not all the free-fall period can be utilized. It will take a finite length of time to determine the direction and amount of desired displacement. In general, the vehicle must be rotated to the desired horizontal attitude after perigee deceleration and must be turned to adjust its azimuth direction. After accelerating horizontally the vehicle must be turned 180° followed by an equal period of deceleration. And finally the vehicle must be rotated from the horizontal to a vertical attitude for the final braking thrust. Various delay times are given in the figure, where delay time includes decision time and the necessary times for the three changes in vehicle orientation. It is apparent that long delay times would destroy the value of this whole procedure.

High perigee altitudes are helpful, since they afford longer free-fall periods in which to utilize the thrust horizontally. High thrust-weight ratios are also desirable for two reasons: (1) Less time need be reserved for the final vertical deceleration, leaving a longer period of free fall (e.g., fig. 4); and (2) higher horizontal acceleration permits moving a greater distance in any given period of time. The figure shows that a high-thrust engine allows very considerable adjustments in landing point, provided that delay time can be held to a minimum. However, the velocity increments for this maneuver, shown in figure 9, are very large for large positional changes. This technique is thus not practical except for making small corrections.

Plane changes. - The method of the preceding section allows positional changes in any direction, even away from the original great circle.

Such changes, however, are of limited extent - at best in the order of tens of miles. It may be desired to execute a landing at a point on the moon far from this great circle. One way to accomplish this is to change the plane of the original circular orbit to a new orbit passing over the desired landing point. Landing may then be carried out by the previously described technique.

The ΔV necessary to change planes is given by

$$\Delta V = 2V_c \sin\frac{\alpha}{2} \tag{B42}$$

This equation may be simplified and written in terms of d, the minimum distance (measured on the lunar surface) between the desired landing point and the original great circle, giving approximately

$$\Delta V \approx 5.86\ d\ (\text{ft/sec}) \tag{B44}$$

where d is in nautical miles. Large values of d are costly to achieve. For example, d of 500 miles requires ΔV of 2900 feet per second. When the latitude of the landing site is less than the inclination of the orbit, another method of making a large positional change is to remain in the initial circular orbit and allow the moon's rotation to effect the change. However, this procedure may take a long time because of the 28-day rotational period of the moon. Whenever a particular landing point is desired, therefore, it is best to approach the moon initially in such a fashion as to permit establishing an orbit that passes over that landing point.

Impact Velocity

It has been pointed out that various errors during the vertical descent may cause the vehicle to strike the ground with a nonzero velocity. The impact velocity can be held to a minimum by placing very exacting requirements on the guidance and controls (aided perhaps with a variable-thrust engine). The difficulties of securing low impact velocities must be balanced against the penalties of designing the vehicle to tolerate higher velocities (stronger structure and/or shock-absorbing devices). As a crude indication of what impact speeds might be acceptable, the following table is presented:

	Impact speed, ft/sec
Very hard airplane landing	$8\frac{1}{2}$
Parachuted Matador missile (with shock-absorbing device)	27
Parachuted Lockheed X-7 test vehicle (with penetration spike)	50
Parachuted instrument capsule (with penetration spike)	55

Note that these values are lower than the tolerable impact speed for a properly supported man (without benefit of shock-absorbing devices), which is 80 feet per second (ref. 8). The limiting impact speed thus seems likely to be fixed by the structural requirements rather than by the human occupants.

E-527

Summary of Velocity Requirements

The following table summarizes the ΔV requirements for a typical lunar landing, starting from a 500,000-foot circular orbit. No energy expenditure is included for correcting the landing point or for establishing the circular orbit:

Maneuver	F/W_m	ΔV, ft/sec
Apogee transfer	2	100
Perigee deceleration at constant altitude (50,000 ft)	3	5750
Vertical deceleration	2	1030
		6880

If the change in kinetic and potential energy between orbit and the surface is divided by $\frac{1}{2}\Delta V^2$, the overall efficiency of the landing is found to be 70 percent. This is comparable to similarly defined efficiencies for satellite-launching trajectories.

CONCLUDING REMARKS

A proposed method for landing on the surface of the moon starting from a high-altitude circumlunar orbit is to transfer to the lowest acceptable altitude by a minimum-energy elliptical path. After braking to a halt, a vertical descent is made consisting of free fall and a final upward thrust application to decelerate the vehicle. Including the initial orbit acquisition, this procedure requires at least four engine starts.

To minimize fuel consumption, the altitude at the perigee of the ellipse should be as low as possible. Factors tending to raise the perigee are the irregular lunar surface, allowances for retrothrust errors at apogee, altitude loss during perigee deceleration, and the ability to correct the landing point during vertical descent.

Perigee deceleration may be accomplished with no altitude loss by proper orientation of the thrust vector. Little reduction in ΔV for this maneuver can be gained by increasing the thrust-weight ratio beyond 2 or 3. Similarly, in the vertical descent, ΔV is prohibitively large for F/W_m of 1, but there is little further reduction from F/W_m over 2. Also, the impact velocity is more sensitive to engine starting errors during vertical descent for high thrust-weight ratios.

Constant-thrust engines will yield acceptable impact velocities if the various errors (starting, trajectory, etc.) can be held sufficiently small. (In a typical case, the thrust-initiation altitude cannot be permitted to vary by more than ± 120 feet in order to limit the impact velocity to 50 feet per second even if no other errors are present.) However, considerably larger errors can be tolerated if the engine possesses some thrust variability. As little as 10-percent variability is helpful, although a two-to-one variation is probably preferable.

If delay times are not too big, corrections in landing point (in the order of a few miles) can be made during the vertical descent. Larger deviations from the original great circle necessitate a change of orbit plane, which requires a large ΔV.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, August 6, 1959

APPENDIX A

SYMBOLS

a	acceleration, ft/sec^2
d	minimum distance (measured on lunar surface) between desired landing point and original great circle, naut. miles
F	engine thrust, lb
g_E	gravitational constant at surface of Earth, 32.174 ft/sec^2
g_m	acceleration due to gravity at lunar surface, 5.32 ft/sec^2
h	altitude, ft
I	specific impulse, sec
m	vehicle mass, slugs
R	maximum horizontal range during vertical descent, ft
r	distance from center of moon, ft
r_m	radius of moon, 5.7×10^6 ft
S	distance measured on lunar surface, ft
t	time, sec
t_{err}	thrust-initiation time error, sec
V	vehicle velocity, ft/sec
ΔV	characteristic-velocity increment, ft/sec
V_c	circular orbit velocity, ft/sec
W_i	initial vehicle weight, lb
W_m	vehicle weight based on gravity at lunar surface, lb
W_p	propellant weight, lb
y	control reaction distance, ft

E-527

E-527

α	angle between orbit planes, deg
β	angle between thrust vector and local horizontal, radians
θ	circumferential angle, radians
μ	gravitational force constant for moon, 1.729×10^{14} ft^3/sec^2

Subscripts:

d	delay
x	start of vertical deceleration
1	apogee
2	perigee
3	at lunar surface

Superscript:

$\cdot$	differentiation with respect to time

APPENDIX B

DERIVATION OF EQUATIONS

Departure from Circular Orbit

The impulsive velocity increment required to transfer from the circular orbit to an elliptical orbit at point 1 in sketch (a) is

$$\Delta V_1 = V_{c,1} - V_1 \tag{B1}$$

where V_1 is the velocity at point 1 on the elliptical orbit. The velocity at any point on an elliptical orbit is given by the following (see, e.g., ref. 9):

$$V = \sqrt{\mu\left(\frac{2}{r} - \frac{2}{r_1 + r_2}\right)} \tag{B2}$$

where $\mu = g_m r_m^2$. Therefore,

$$V_1 = \sqrt{\frac{2\mu r_2}{r_1(r_1 + r_2)}} \tag{B3}$$

The circular velocity of an orbit can be expressed as

$$V_c = \sqrt{\mu/r} \tag{B4}$$

Thus, from equations (B1), (B3), and (B4),

$$\Delta V_1 = V_{c,1}\left(1 - \sqrt{\frac{2}{\frac{r_1}{r_2} + 1}}\right) \tag{B5}$$

Perigee Maneuver

Impulsive thrust. - At point 2 of sketch (a), or perigee, the vehicle velocity is arrested. Therefore,

$$\Delta V_2 = V_2 - 0 \tag{B6}$$

From equation (B2),

$$\Delta V_2 = V_{c,2}\sqrt{\frac{2}{\frac{r_2}{r_1} + 1}} \tag{B7}$$

E-527

Continuous circumferential thrust. - An approximate solution was made of the differential equations of motion, similar to the method presented in reference 10. The differential equations for motion in the radial and circumferential directions are

$$\ddot{r} = r\dot{\theta}^2 - \frac{\mu}{r^2} + \frac{F}{m} \sin \beta \tag{B8}$$

$$\frac{d}{dt} (r^2\dot{\theta}) = -r \frac{F}{m} \cos \beta \tag{B9}$$

where β is held equal to zero. In general, the velocity at the start of the maneuver will be somewhat higher than the circular orbit velocity at the perigee altitude. However, for simplicity, this excess velocity is ignored, and the initial conditions are

$$\left.\begin{array}{ll} r = r_2 & \ddot{r} = 0 \\ \dot{r} = 0 & \dot{\theta} = \dfrac{\sqrt{\mu}}{r_2^{3/2}} \end{array}\right\} \text{at} \quad t = 0 \tag{B10}$$

Eliminating $\dot{\theta}$ from equation (B9) by substituting from equation (B8),

$$\frac{d}{dt} \sqrt{r^3\ddot{r} + \mu r} = -r \frac{F}{m} \tag{B11}$$

For large values of F/m, the deceleration time will be small and little change in radial position will occur. Therefore, $r \approx r_2$ for any t, and

$$\frac{d}{dt} \sqrt{r_2^3\ddot{r} + \mu r_2} = -r_2 \frac{F}{m} \tag{B12}$$

Integrating and applying the initial conditions yield

$$\sqrt{r_2^3\ddot{r} + \mu r_2} = -r_2 \left(\frac{F}{m}\right)t + \sqrt{\mu r_2} \tag{B13}$$

taking F/m as constant. Solving for $\ddot{r}$ gives

$$\ddot{r} = \frac{(F/m)^2 t^2}{r_2} - \frac{2\sqrt{\mu r_2}(F/m)t}{r_2^2} \tag{B14}$$

The time required to arrest the perigee motion is found by combining equations (B8) and (B14) and setting $\dot{\theta} = 0$, giving

$$t = \frac{\sqrt{\mu/r_2}}{F/m} \tag{B15}$$

Or, in terms of the thrust-weight ratio at the lunar surface, the time required to arrest the perigee velocity is

$$t = \frac{\sqrt{\mu/r_2}}{(F/W_m)g_m} \tag{B16}$$

The radial velocity of the vehicle is obtained by integrating equation (B14):

$$\dot{r} = \frac{(F/m)^2 t^3}{3r_2} - \frac{\sqrt{\mu r_2}(F/m)t^2}{r_2^2} \tag{B17}$$

where, from the initial conditions, the constant of integration is zero. The radial velocity at the instant the circumferential deceleration is completed is found by substituting equation (B16) into equation (B17):

$$\dot{r} = \frac{-2\sqrt{\mu}\ r_m^2}{3\left(\frac{F}{W_m}\right)r_2^{5/2}} \tag{B18}$$

The radial position at any time is obtained by integrating equation (B17):

$$r = \frac{(F/m)^2 t^4}{12r_2} - \frac{\sqrt{\mu r_2}(F/m)t^3}{3r_2^2} + r_2 \tag{B19}$$

At the completion of circumferential deceleration, the radial position is found by combining equations (B15) and (B19), giving

$$r = r_2 - \frac{r_m^4}{4r_2^3(F/W_m)^2} \tag{B20}$$

The decrease in altitude is $(r_2 - r)$. Thus,

$$\Delta h = \frac{r_m^4}{4r_2^3(F/W_m)^2} \tag{B21}$$

which is noted to be a small quantity relative to r_2 for F/W_m greater than about 3, thus confirming the assumption of equation (B12).

The change in circumferential position during the perigee maneuver can be found by integrating equation (B8):

$$\dot{\theta}^2 = \frac{1}{r^3}(r^2\ddot{r} + \mu) \tag{B22}$$

E-527

E-527

CJ-3 back

Substituting $\ddot{r}$ from equation (B14) into equation (B22) and again assuming that $r \approx r_2$ yield

$$\dot{\theta} = \frac{1}{r_2^{3/2}} \left[\sqrt{\mu} - \sqrt{r_2}(F/m)t\right] \tag{B23}$$

Integration of equation (B22) gives

$$\theta = \frac{1}{r_2^{3/2}} \left[\sqrt{\mu}t - \frac{\sqrt{r_2}(F/m)t^2}{2}\right] \tag{B24}$$

where, from the initial conditions, the constant of integration is zero. The circumferential angle at the completion of perigee deceleration can be obtained by substituting t from equation (B15) into equation (B24):

$$\theta = \frac{\mu}{2r_2^2(F/m)} \tag{B25}$$

The circumferential distance, measured along the lunar surface, through which the vehicle travels is

$$S = r_m\theta \tag{B26}$$

Thus, in terms of the thrust-weight ratio,

$$S = \frac{r_m^3}{2r_2^2(F/W_m)} \tag{B27}$$

For low values of F/W_m (e.g., <3), more exact results can be obtained by the methods discussed in reference 10. However, equations (B16), (B18), (B21), and (B27) are sufficiently accurate for the range of F/W_m shown in figure 2. (As a check, eqs. (B8) and (B9) were integrated numerically and yielded results at F/W_m of 6 that differed by less than 0.5 percent.)

Constant-altitude deceleration. - In the procedure for constant-altitude deceleration, the thrust vector is gradually tilted away from its initial horizontal direction so that its vertical component plus centrifugal force always equal the vehicle weight. Mathematically, this corresponds to setting $\dot{r}$ and $\ddot{r}$ equal to zero in equations (B8) and (B9), yielding

$$\dot{\theta}^2 = \frac{\mu}{r^3} - \frac{F}{mr}\sin\beta \tag{B28}$$

$$\ddot{\theta} = -\frac{F}{mr}\cos\beta \tag{B29}$$

The required variation of β is obtained by differentiating equation (B28) and combining with equation (B29), giving simply

$$\beta = 2\theta \tag{B30}$$

Substituting equation (B30) in equation (B28) gives

$$\left.\begin{aligned} \dot{\theta} &= \sqrt{\frac{\mu}{r^3} - \frac{F}{mr} \sin 2\theta} \\ t &= \int_0^{\theta_1} \frac{d\theta}{\sqrt{\frac{\mu}{r^3} - \frac{F}{mr} \sin 2\theta}} \end{aligned}\right\} \tag{B31a}$$

where the upper limit is obtained by setting $\dot{\theta} = 0$,

$$\sin 2\theta_1 = \frac{\mu/r^3}{F/mr} = \frac{\mu/r^2}{F/m}$$

After substituting $\varphi = \sin 2\theta$ in equation (B31a), a solution is found in terms of an elliptic integral of the first kind (e.g., ref. 11, number 546):

$$t = \frac{1}{\sqrt{2b}} \operatorname{sn}^{-1} \left(\sqrt{\frac{2}{1 + b}}, \sqrt{\frac{1 + b}{2b}} \right) \tag{B31b}$$

where $1/b = \sin 2\theta_1$. Then ΔV may be obtained by

$$\Delta V = g_m \left(\frac{F}{W_m}\right) t \tag{B32}$$

Vertical Descent

If the vehicle is to have no impact velocity upon landing, the velocity gained during the free-fall portion of the descent must be equal to that which can be arrested during the powered phase. Therefore,

$$V_x^2 = 2g_m(h_2 - h_x) = 2ah_x \tag{B33}$$

assuming g is constant. Thus, the altitude at which the thrust must be applied is

$$h_x = \frac{h_2}{\frac{a}{g_m} + 1} \tag{B34}$$

The acceleration during the powered phase of the descent is found from a force analysis of the vehicle:

$$F - W_m = ma \tag{B35}$$

$$a = \left(\frac{F}{W_m} - 1\right) g_m \tag{B36}$$

The free-fall flight time is

$${}_{2}\Delta_{x} t = \frac{V_x}{g_m} \tag{B37}$$

where ${}_{2}\Delta_{x}$ signifies $t_2 - t_x$. The time for the powered phase is

$${}_{x}\Delta_{3} t = \frac{V_x}{a} \tag{B38}$$

and ΔV is again given by equation (B32).

Correction of Horizontal Position

The maximum horizontal distance through which the vehicle can travel during the free-fall portion of the vertical descent with the stipulation that the horizontal velocity at the end of free fall be zero is

$$R = 2\left[\frac{1}{2}\, a \left(\frac{{}_{2}\Delta_{x} t - t_d}{2}\right)^2\right] \tag{B39}$$

The horizontal acceleration of the vehicle is

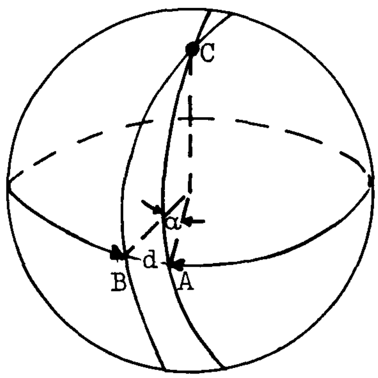
$$a = \frac{F}{W_m}\, g_m \tag{B40}$$

Thus,

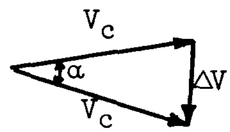
$$R = \frac{F}{W_m}\, g_m \left(\frac{{}_{2}\Delta_{x} t - t_d}{2}\right)^2 \tag{B41}$$

Plane Change

In sketch (b), the point A represents the desired landing point, and line BC is the original great circle whose point of nearest approach to A is B. The great circle distance between B and A is denoted by d. In order to land at A, the plane of the initial orbit must be changed through an angle α to the new orbit indicated by line AC. The plane change must be effected at point C, the node of the two orbits, which is 90° before the desired landing point for minimum energy expenditure.



(b)



(c)

As indicated by the velocity-vector diagram (sketch (c)), the original velocity vector V_c at point C must be increased by a vector ΔV such that the magnitude of the resultant is still V_c, but the direction is changed by an angle α. By geometry, then, the required ΔV for the plane change is

$$\Delta V = 2V_c \sin \frac{\alpha}{2} \tag{B42}$$

If α is small, $\sin (\alpha/2) \approx \alpha/2$. Also, $d = r_m \alpha$. Thus,

$$\Delta V \approx \frac{V_c d}{r_m} \tag{B43}$$

For orbits close to the lunar surface, a general value of $V_c = 5500$ feet per second can be substituted in equation (B43):

$$\Delta V \approx \frac{5500}{5.7(10^6)} (6076)d$$

or

$$\Delta V \approx 5.86\ d\ (\text{ft/sec}) \tag{B44}$$

where d is in nautical miles.

REFERENCES

1. von Braun, Wernher, Whipple, Fred L., and Ley, Willy (Cornelius Ryan, ed.): Conquest of the Moon. Viking, New York, 1953.

2. Clement, George H.: The Moon Rocket. P-833, The RAND Corp., May 1956.

3. Ehricke, Krafft A.: On the Mechanics of Descent to a Celestial Body. Jour. Astronautics, vol. 2, no. 4, 1955, pp. 137-144.

4. Lang, H. A.: Lunar Instrument Carrier - Landing Factors. RM-1725, The RAND Corp., June 4, 1956.

5. Cross, C. A.: An Analogue Computer for the Vertical Rocket Landing and Take Off Problems. Jour. British Interplanetary Soc., vol. 15, no. 1, Jan.-Feb. 1956, pp. 7-17.

6. Wilkins, H. Percy: Where to Land on the Moon. Jour. British Interplanetary Soc., vol. 13, no. 2, Mar. 1954, pp. 65-67.

7. Smith, R. A.: Landing on Airless Planets. Jour. British Interplanetary Soc., vol. 12, no. 4, July 1953, pp. 149-155.

8. Kornhauser, Murray: Impact Protection for the Human Structure. Proc. Am. Astronautical Soc., Western Regional meeting, Palo Alto (Calif.), Aug. 18-19, 1958, pp. 38-1 to 38-9.

9. Synge, J. L., and Griffith, B. A.: Principles of Mechanics. McGraw-Hill Book Co., Inc., 1942, p. 178.

10. Tsien, H. S.: Take-Off from Satellite Orbit. Jour. Am. Rocket Soc., vol. 23, no. 4, July-Aug. 1953, pp. 233-236.

11. Peirce, B. O.: A Short Table of Integrals. Third ed., Ginn and Co., 1929.

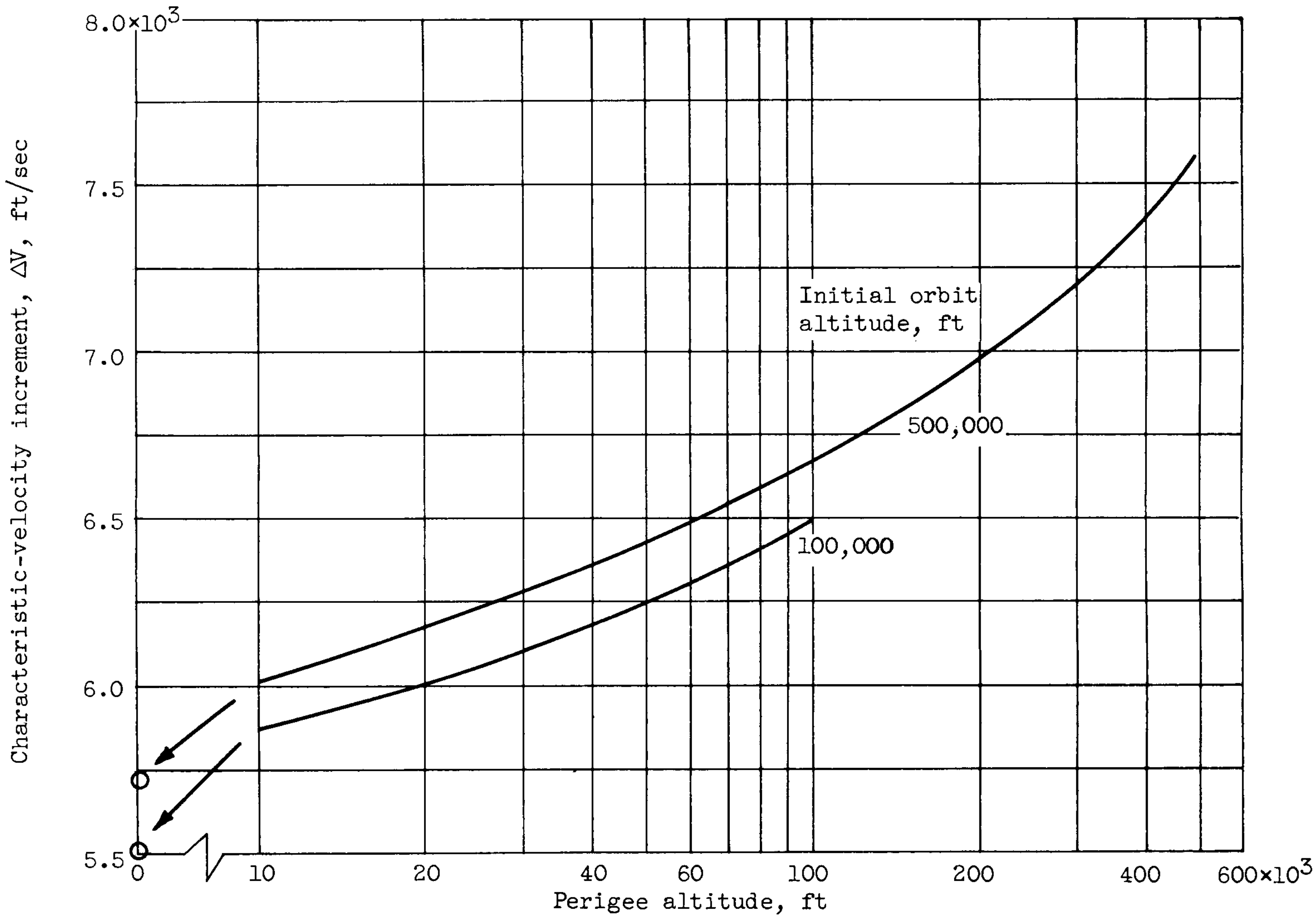
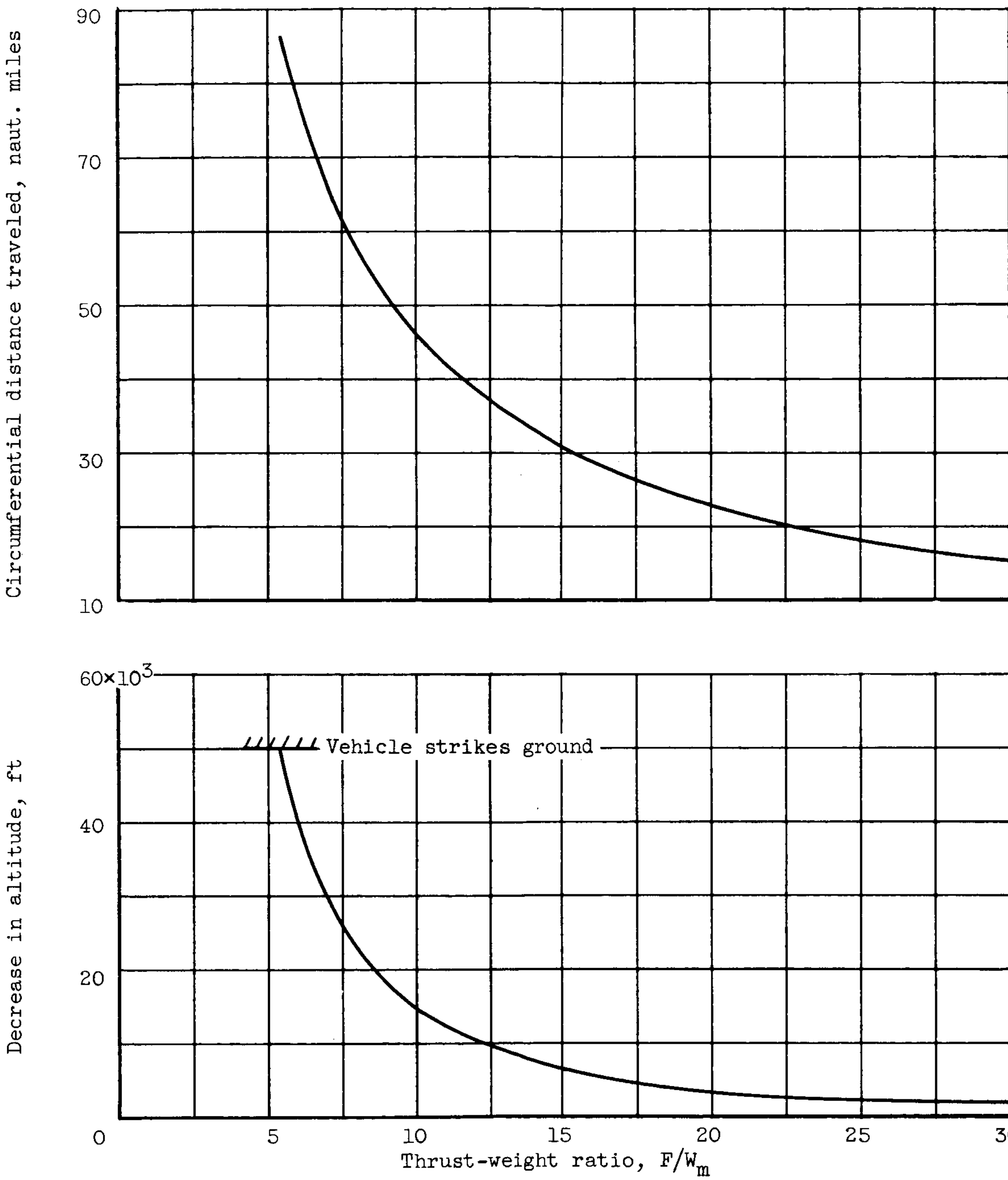


Figure 1. - Benefit of minimum-energy transfer to lower altitude, followed by vertical descent. Impulsive thrust.

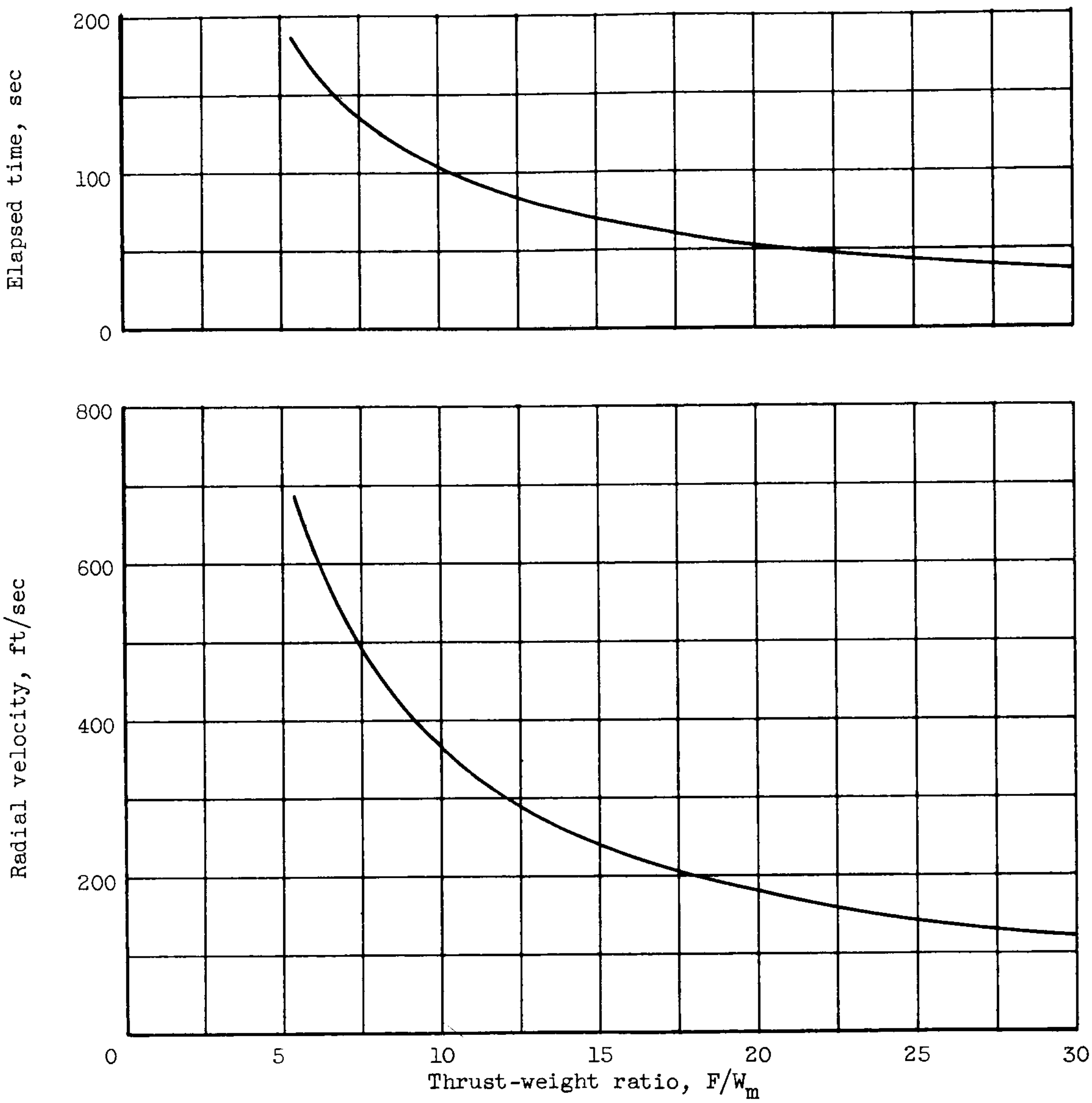
E-527

CL-4



(a) Change in position.

Figure 2. - Conditions at completion of perigee deceleration. Circumferential thrust; deceleration started at perigee altitude of 50,000 feet.



(b) Elapsed time and radial velocity.

Figure 2. - Concluded. Conditions at completion of perigee deceleration. Circumferential thrust; deceleration started at perigee altitude of 50,000 feet.

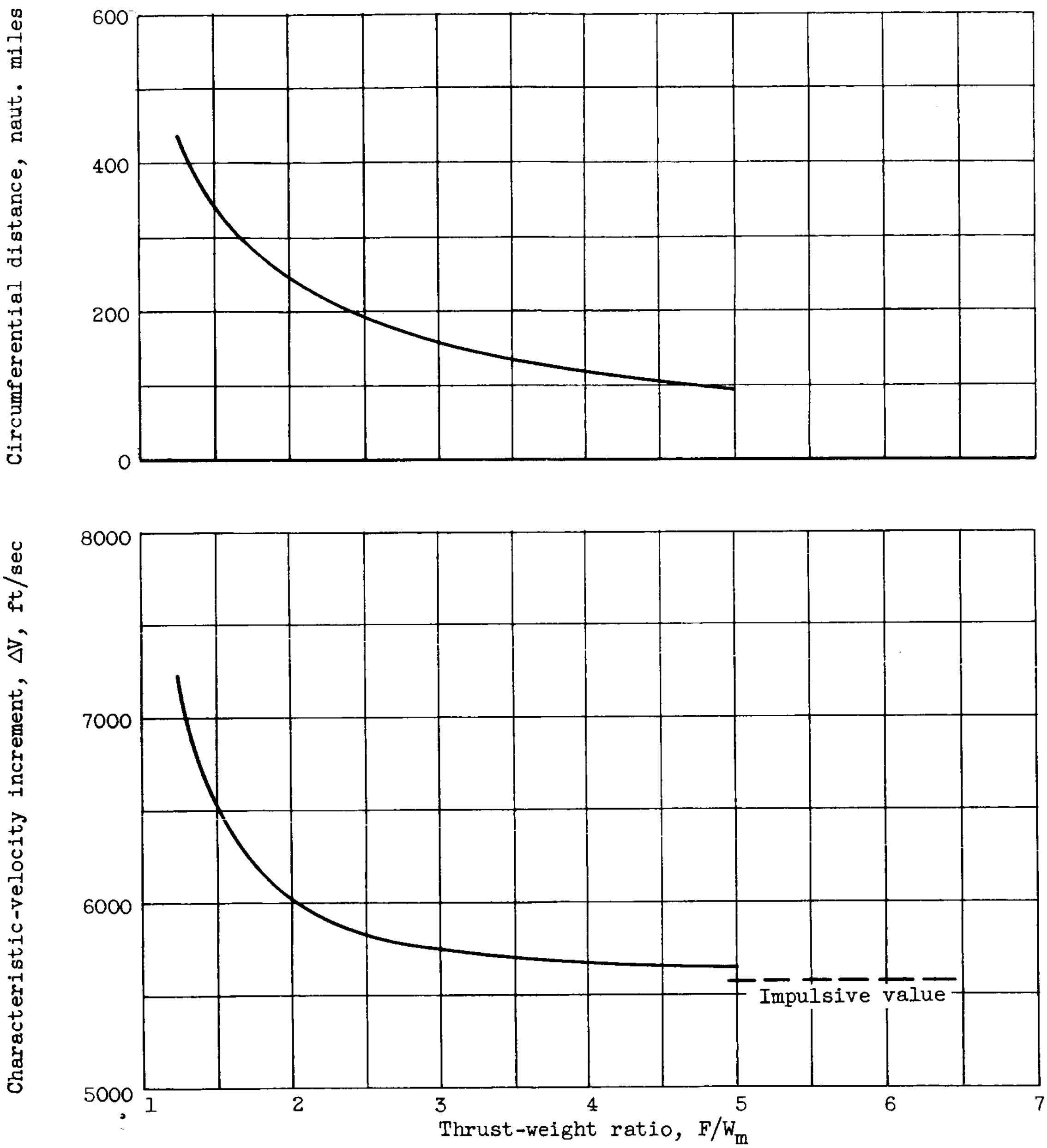
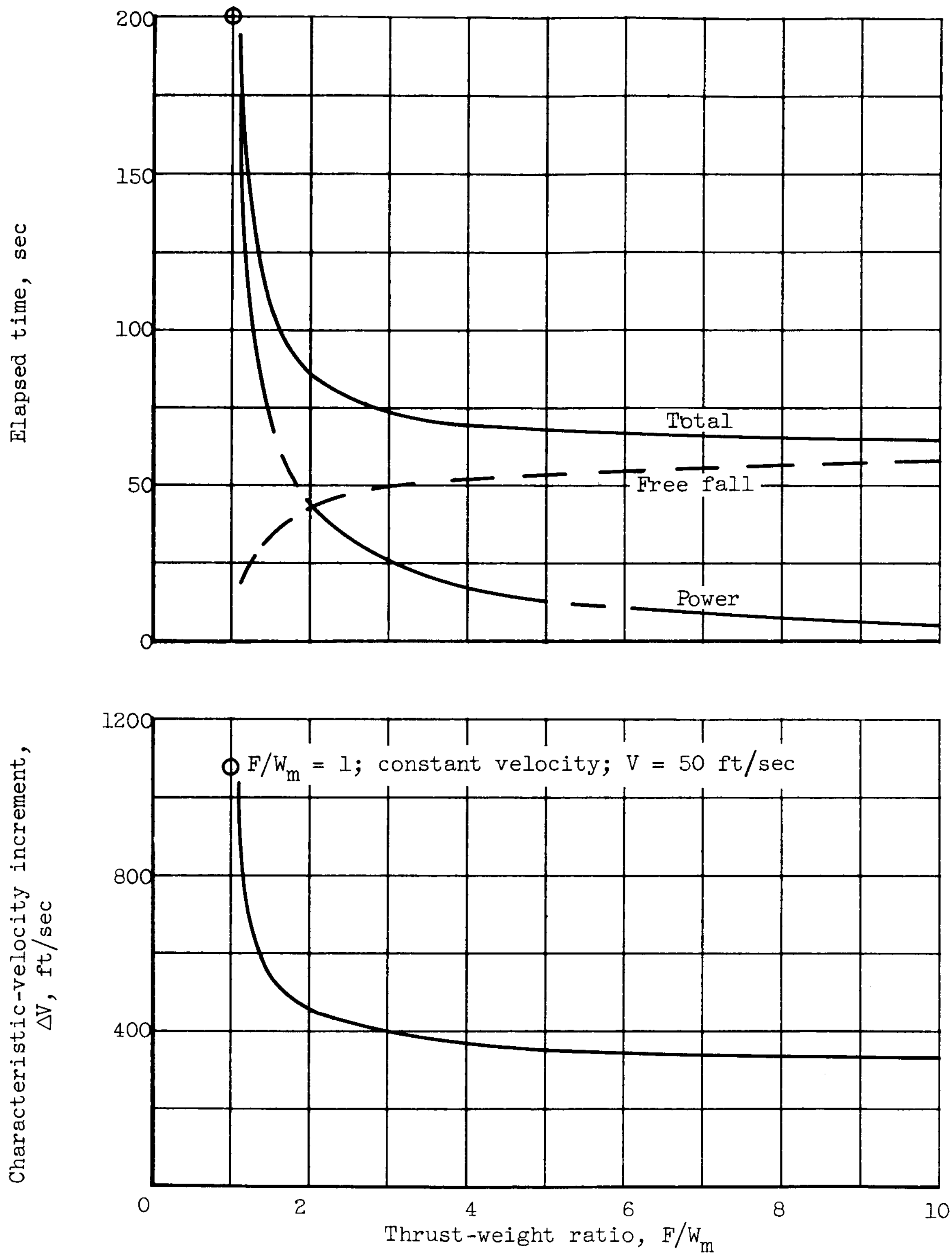


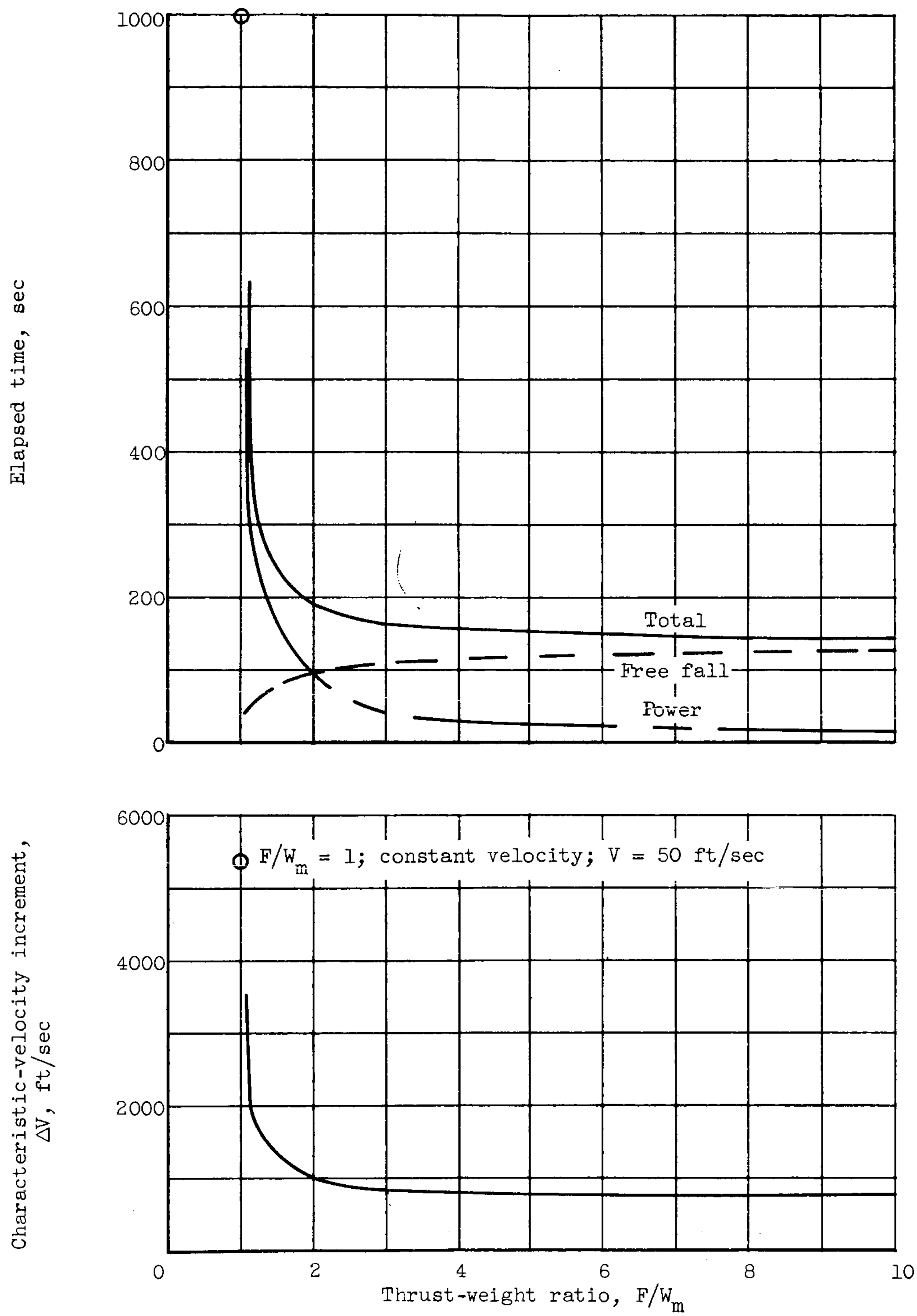
Figure 3. - Circumferential displacement and velocity increment required for perigee deceleration at constant altitude of 50,000 feet.



(a) Perigee altitude, 10,000 feet.

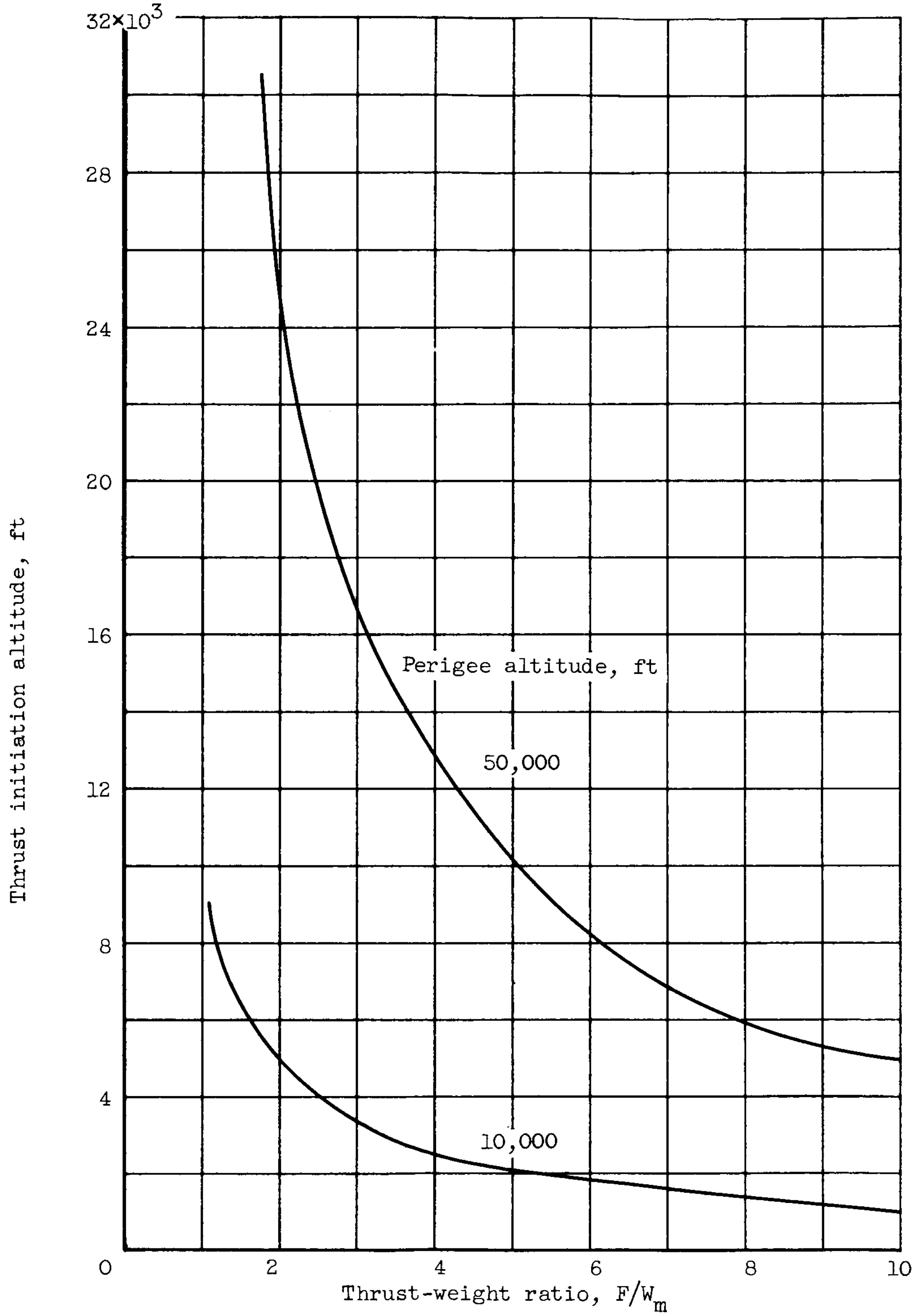
Figure 4. - Effect of thrust level during vertical descent.

E-527



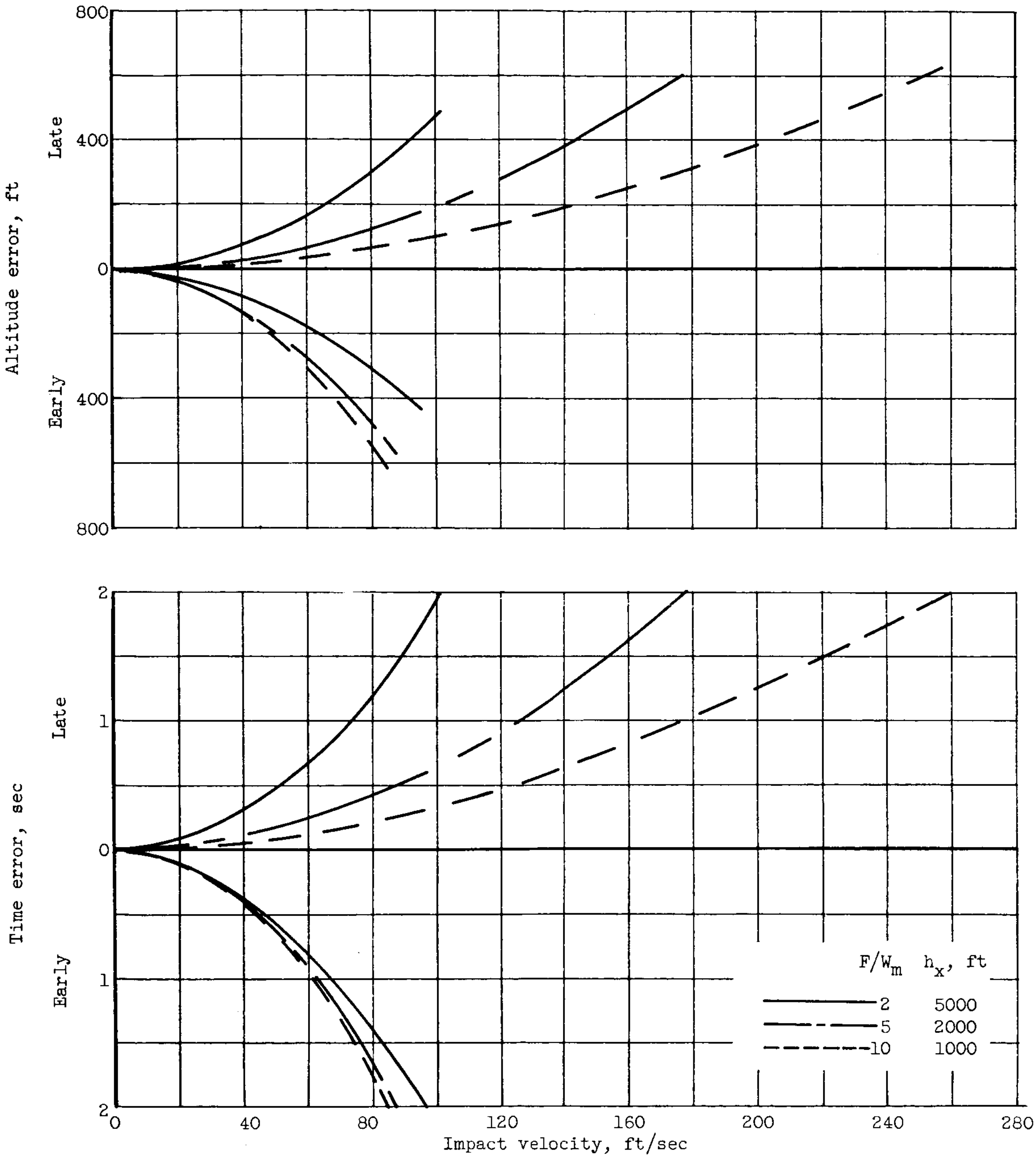
(b) Perigee altitude, 50,000 feet.

Figure 4. - Continued. Effect of thrust level during vertical descent.



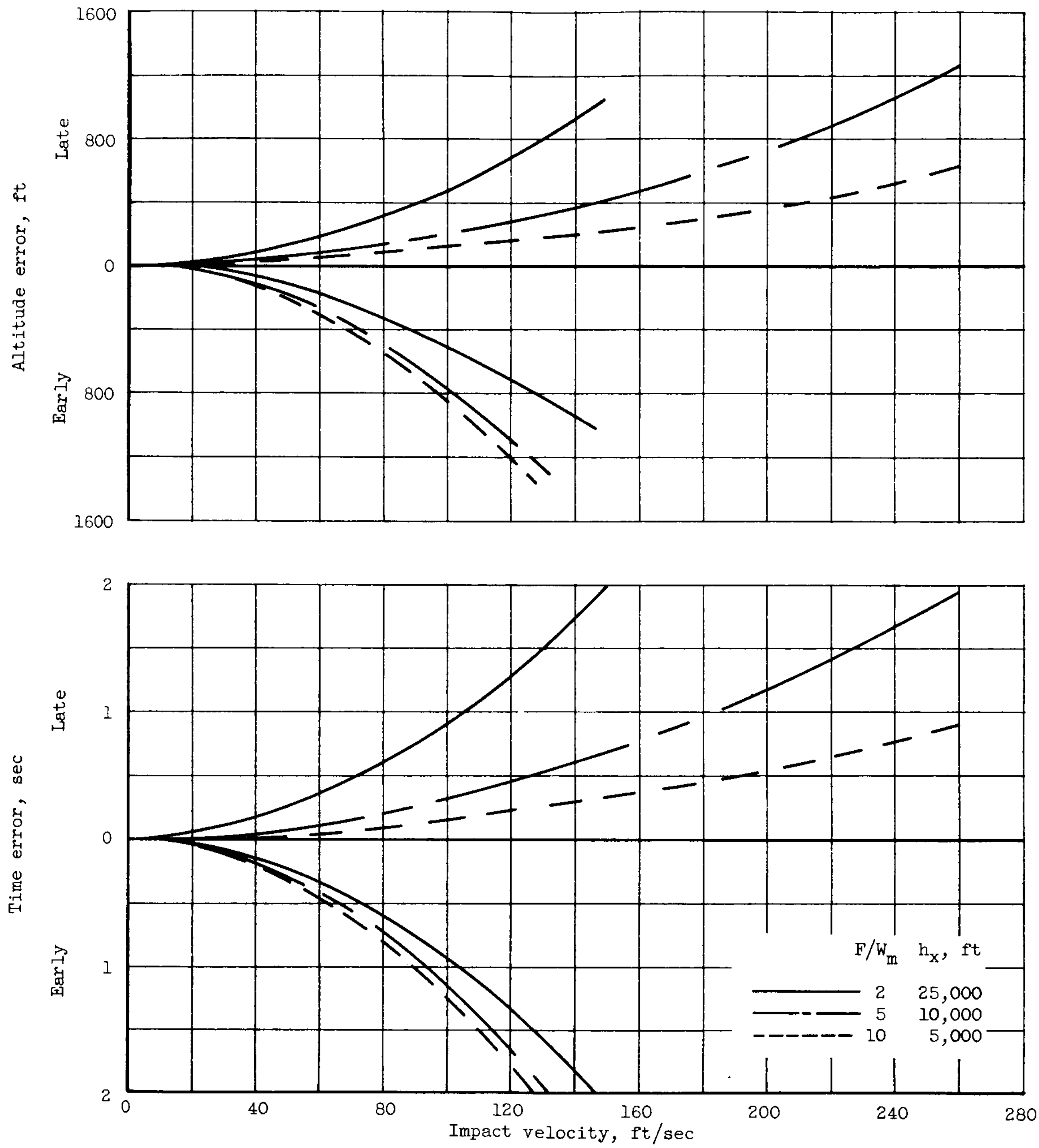
(c) Thrust initiation altitude.

Figure 4. - Concluded. Effect of thrust level during vertical descent.



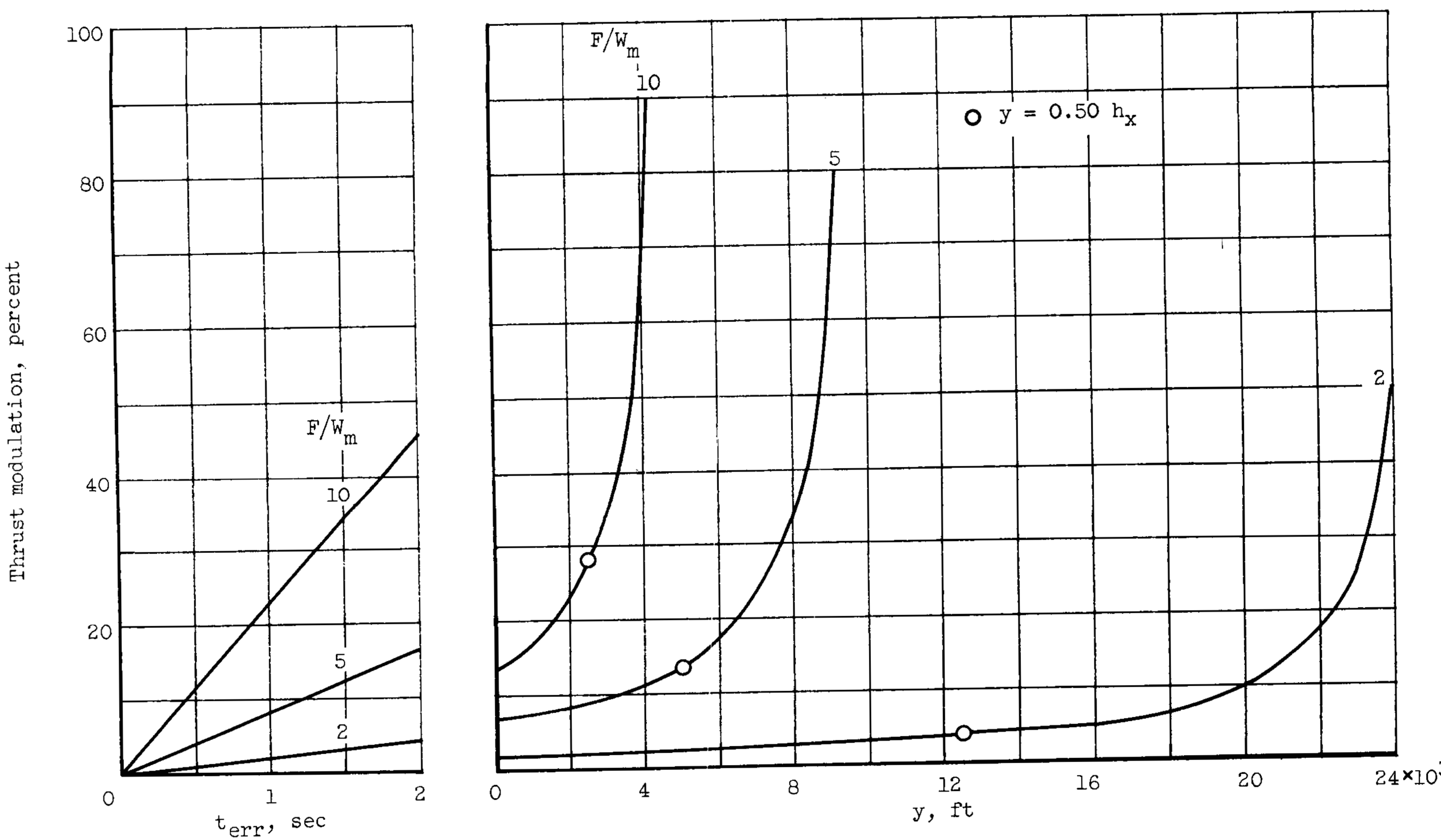
(a) Perigee altitude, 10,000 feet.

Figure 5. - Effect of error in starting engine during vertical descent.



(b) Perigee altitude, 50,000 feet.

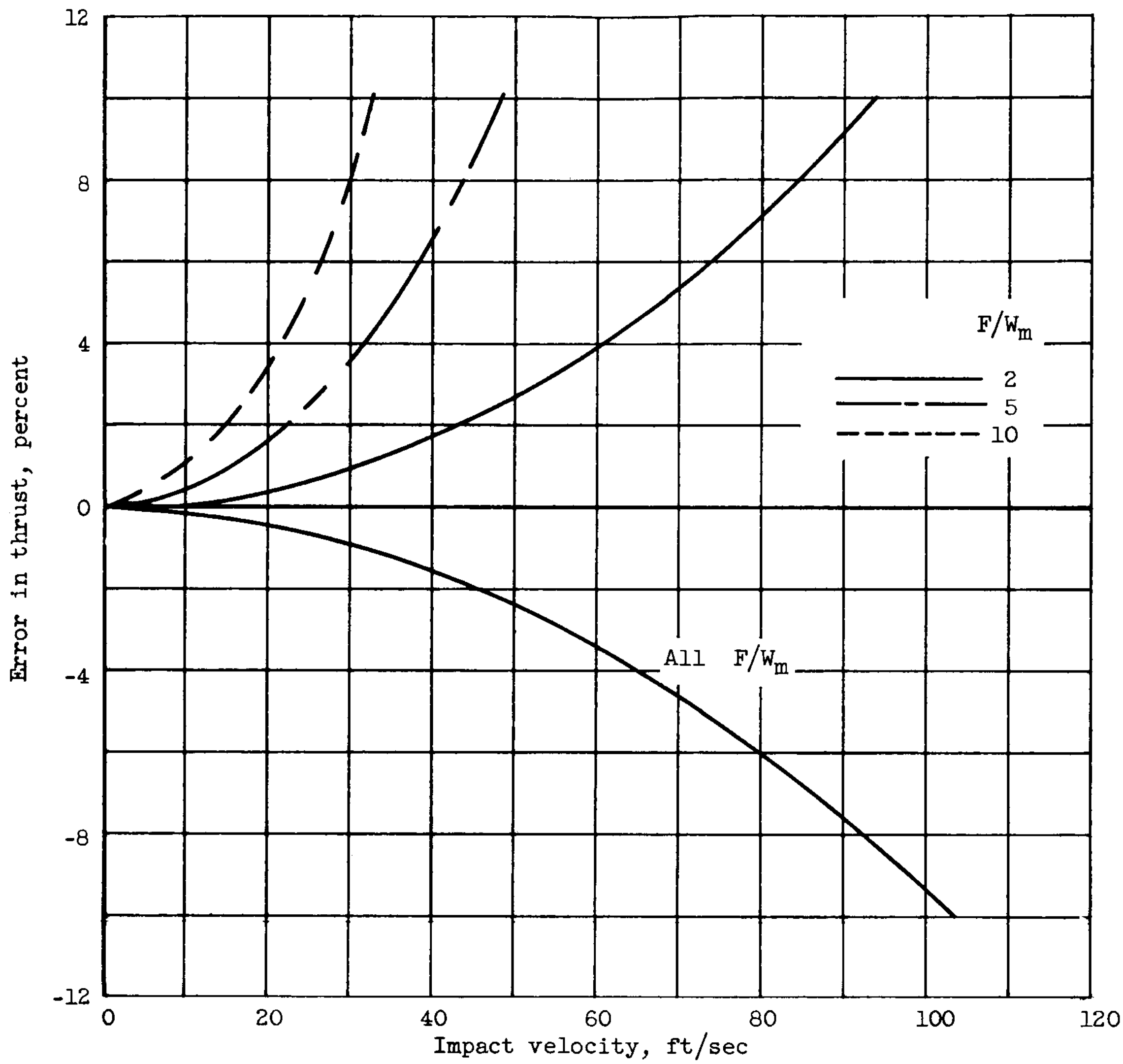
Figure 5. - Concluded. Effect of error in starting engine during vertical descent.



(a) Time error. Control reaction distance, 2000 feet.

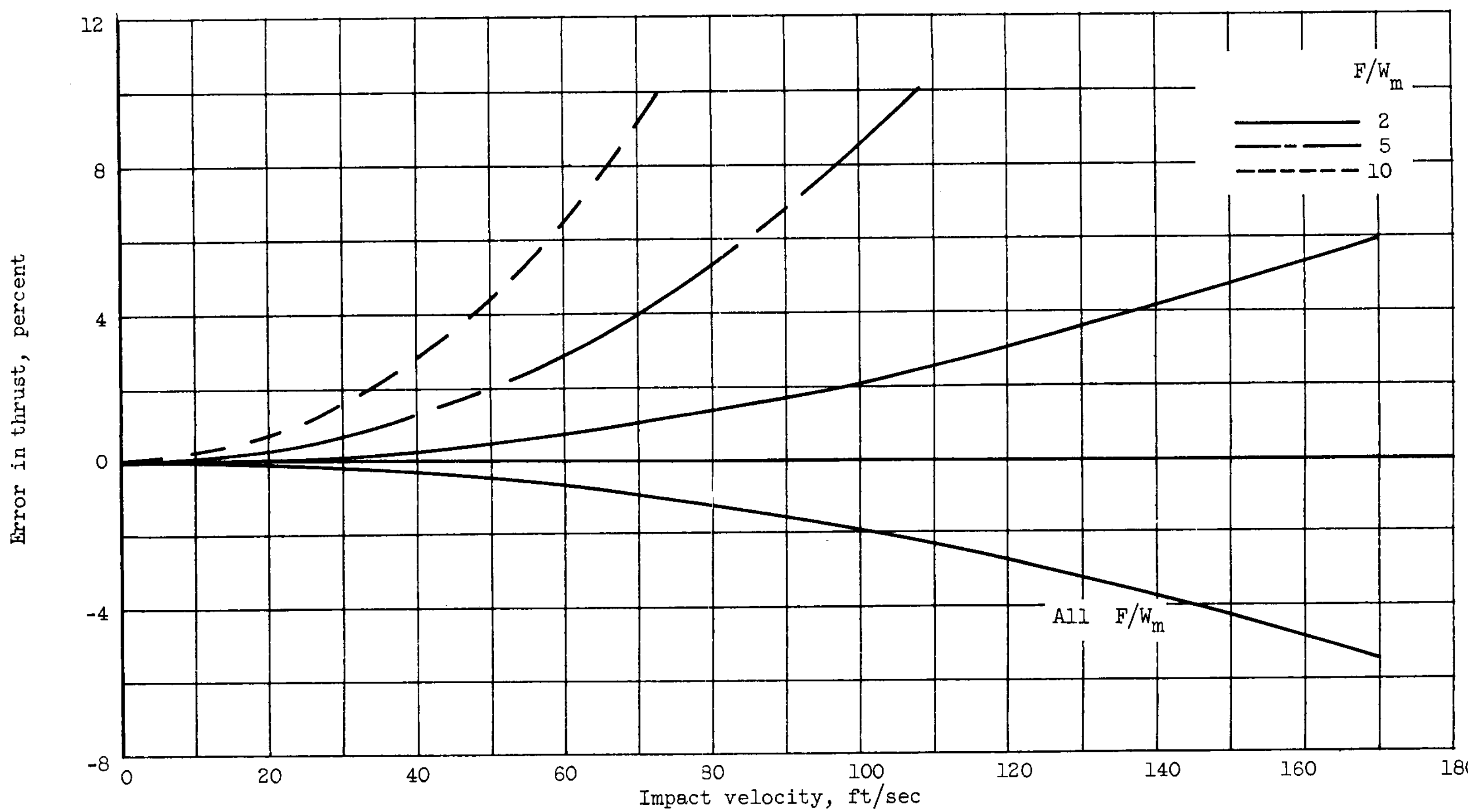
(b) Control reaction distance. Time error, 1 second (early).

Figure 6. - Thrust modulation required for zero impact velocity, for vertical descent from 50,000-foot altitude.



(a) Perigee altitude, 10,000 feet.

Figure 7. - Effect of error in thrust level during vertical descent.



(b) Perigee altitude, 50,000 feet.

Figure 7. - Concluded. Effect of error in thrust level during vertical descent.

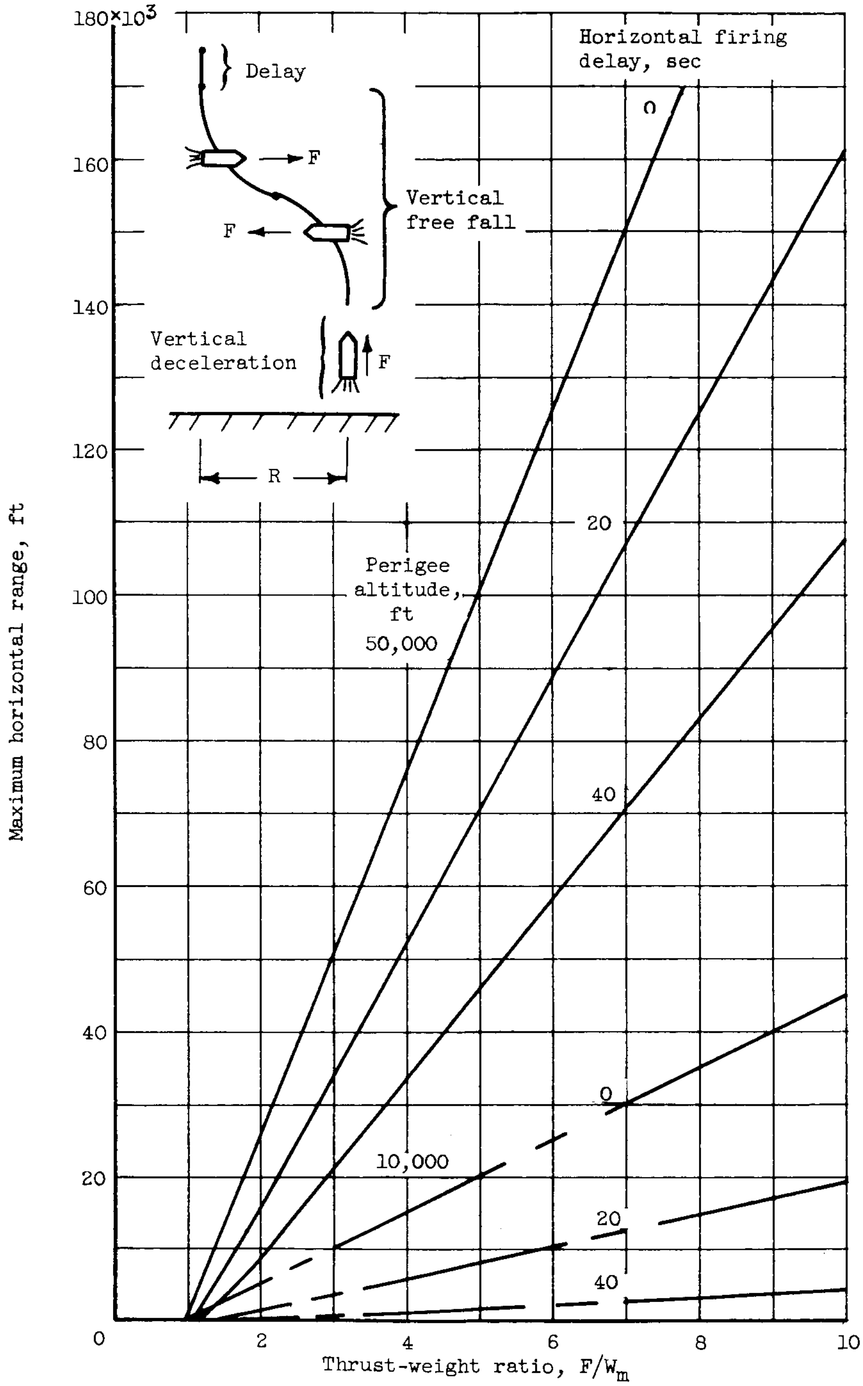


Figure 8. - Maximum horizontal range during vertical descent.

E-527

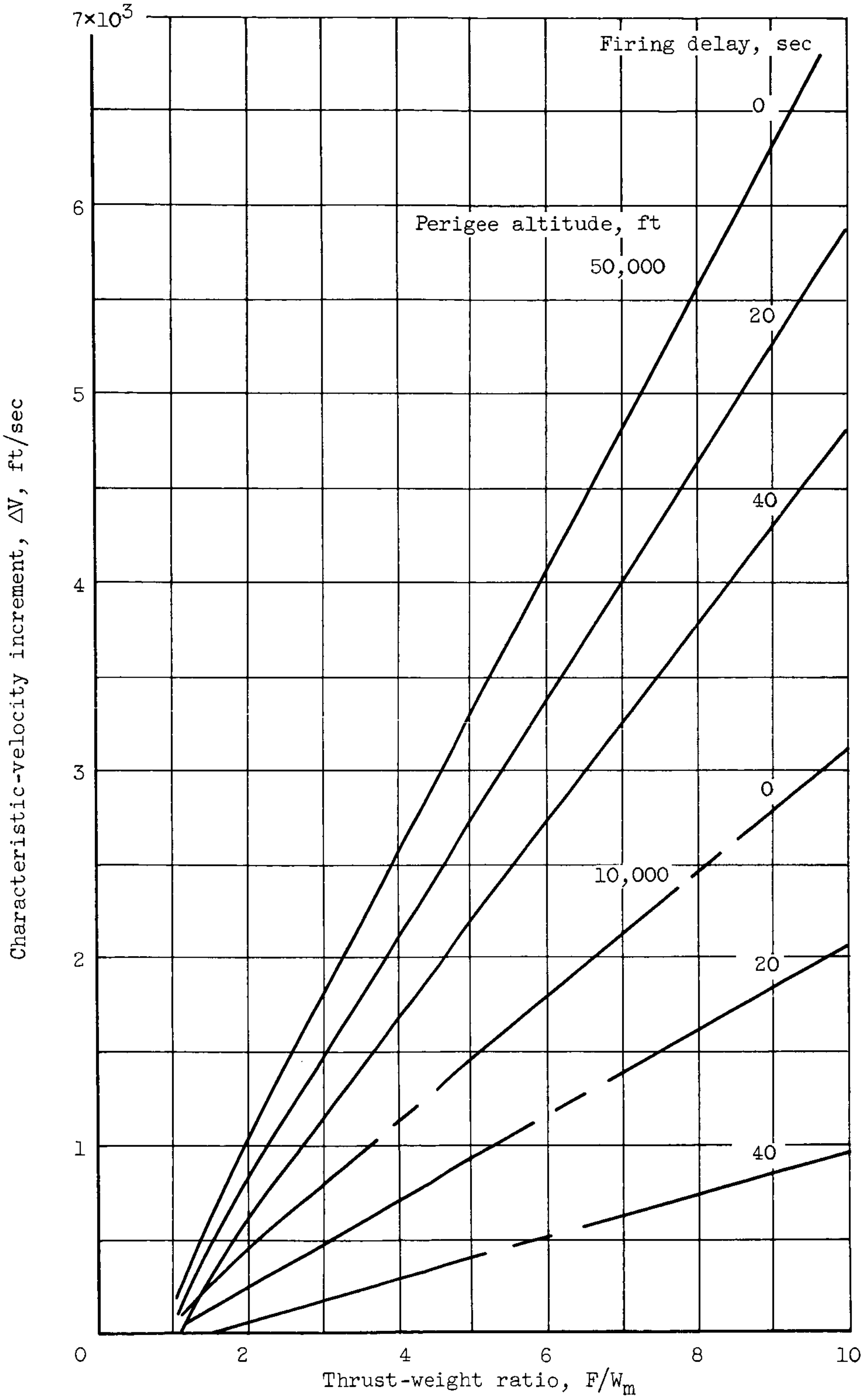


Figure 9. - Velocity increment required for maximum horizontal travel during vertical descent.